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LIFE SPAN OF RESONANTING RAPIDLY OSCILLATING SHOCK WAVES

Abstract. This paper is concerned with nonlinear geometric optics for strictly hyperbolic systems of conservation laws in one space variable. Recent work of the author on small amplitude, rapidly oscillating perturbations of shock waves is reviewed and the case of the Euler system in gasdynamics is treated in detail. A general result about the validity in the large of the asymptotics is given.

Introduction

Under the term of "geometric optics" it is currently meant a set of techniques providing some asymptotic expansions for solutions of (systems of) partial differential equations. This method, also called W.K.B. method, or stationary phase method, originated in physical environment in the first half of this century. In the mathematical milieu it was used probably for the first time by Lax [L] (1957) to solve hyperbolic linear systems of PDE's with highly oscillating initial data $a(x)e^{i\varphi_0(x)/\varepsilon}$, ε a small positive parameter, $x \in R^n$. Solutions were sought under the form

$$u_\varepsilon(x,t) \sim e^{i\varphi(t,x)/\varepsilon}\left(u_0(t,x) + \varepsilon u_1(t,x) + \ldots\right);$$

there appeared all the essential ingredients of the method: the eikonal equation for the phase function φ, the polarization of the zero-order term along right eigenvectors, the equations for the correcting terms. This method was soon exploited in a variety of linear problems; let us just quote the proof of unsolvability of some partial differential operators in [H] (1960).

The first step toward nonlinear geometric optics was taken by Choquet-Bruhat [CB1] (1969), who made formal computations in the case of quasilinear systems. The problem got a new impulse in the eighties thanks to Hunter and Keller [HK] (1982) and then to some papers of Hunter, Majda and Rosales [MR], [HMR], where multiphase developments were introduced and the related resonance conditions clarified. Unfortunately, all of these papers provided only formal results. The justification of the theory was proved recently in a series of papers by Joly, Métivier and Rauch (see [J], [JMR1], [JMR2] and references there; see also [G] and [Sc]). For an introduction to these problems the reader is referred to [W], [CB2], [M2]; for a different point of view see [Y].

Let us be a little more precise. We consider a quasilinear strictly hyperbolic system in one space dimension,

$$\partial_t u + A(u)\partial_x u = 0,$$

where $t, x \in R$, u is R^N-valued and A is a smooth $N \times N$ matrix. Let u_0 be a smooth solution of this system (called the background state) and set $u_{|_{t=0}} = h_0$. Then let us consider the same equations but coupled with some perturbed initial conditions, say $u_{|_{t=0}} = h_0 + \varepsilon h_\varepsilon$, where ε is a small positive parameter as above. On h_ε we assume that it is of the form

$$h_\varepsilon(x) = H\left(x, \frac{\vec{\varphi}_0(x)}{\varepsilon}\right) + o(1)$$

for some smooth function $H(x, \theta)$, almost periodic in the fast θ variables; $o(1)$ means terms converging to 0 in L^∞ norm as $\varepsilon \to 0$. We are thus faced to some problems, that characterize weakly nonlinear geometric optics. Can we find a solution under the form $u = u_0 + \varepsilon u_\varepsilon$ defined in a domain independent of ε, at least for ε sufficiently small? Can we approximate the term u_ε by almost periodic rapidly oscillating functions, say, $u_\varepsilon(t,x) = U\left(t, x, \vec{\varphi}(t,x)/\varepsilon\right) + o(1)$, and what kind of equations must U satisfy? How are determined the function phases $\vec{\varphi}$, and how do they interact? A motivation to these problems comes clearly from high frequency physics; on the other hand, from a numerical point of view, it is sometimes easier to compute the profiles than the exact solutions.

Some definitive answers to these questions are given in [JMR1]. A striking difference with respect to the linear case is the interaction of phase functions. While in the linear case phases mutually cross without interacting, in the presence of nonlinearities many new features appear: creation of new phases, phase shifts, and, more interestingly, resonances. This last phenomenon occurs when phases associated to different fields (eigenvalues) become linearly dependent over Z.

Recently the author has considered the case of a (curved) shock wave as background state ([C]). This means that the former u_0 is a weak solution to a system of conservation laws (in the preceding notations A is therefore the Jacobian matrix of some flux function f); such a solution is discontinuous along an unknown noncharacteristic curve, called the shock curve. Some formal calculations concerning weakly nonlinear geometric optics for this problem were made previously in [MA]. In the case under consideration the matter is somewhat more involved in comparison to the smooth Cauchy problem up here, owing to the reflexion-transmission conditions on the shock curve (the Rankine-Hugoniot conditions). Moreover, extra phases on the shock curve are needed, and new phenomena concerning the perturbation of the shock curve are observed.

In the present paper we carry on the analysis of the asymptotics by precising how the life spans of the exact solutions are connected to those of the profiles; in other words, how long lasts the asymptotics. We begin in Section 1 by giving some accounts on the results of

[C]. They are presented here in a less general version both for sake of brevity and since the theorem on the life span actually requires some stronger assumptions. Both for lack of space and in order to avoid repetitions, only the main definitions are given here; for many more details and examples the reader is referred to [C]. On the other hand, we give in Section 2 an example regarding one-dimensional gasdynamics, with a special emphasis to the boundary conditions for the problem of the profiles. The proof of Theorem 1.2 is given in the general case (by admitting phase shifts, too, and then weakening the assumption of closure), but for the above reasons it has been somewhat compressed. It is contained in the last Section 3.

1. Rapidly oscillating shock waves

We deal with the following system of N conservation laws with source terms in one space dimension:

$$\partial_t u + \partial_x f(u) = b(t, x, u). \tag{1.1}$$

Here t and x are real variables and the functions u, f, b are R^N-valued, $N \geq 1$. Let O be an open subset of R^N, D an open subset of the half plane $\{(t,x) \in R^2; t \geq 0\}$ such that $D \cap \{t = 0\}$ is an interval having 0 as interior point. We assume that $f \in C^2(O)$ and $b \in C^1(D \times O)$. We denote by $A = A(u)$ the Jacobian matrix of the function f and assume that $A(u)$ has N real distinct eigenvalues

$$\lambda_1(u) < \lambda_2(u) < \ldots < \lambda_N(u)$$

for every $u \in O$. Let $\{r_1(u), \ldots, r_N(u)\}$ be a basis of right eigenvectors of $A(u)$, and let $\{l_1(u), \ldots, l_N(u)\}$ be a basis of left ones; we may assume that they are normalized in such a way that $l_j r_k = \delta_{jk}$.

Let λ_κ be a genuinely nonlinear eigenvalue, for some κ (greek kappa) between 1 and N; this means that $\nabla\lambda_\kappa(u) \cdot r_\kappa(u) \neq 0$, for every $u \in O$. We may therefore consider a shock wave (u_0, p_0) attached to it. Let us recall what we mean by this (see [Sm]). The function p_0 belongs to $C^2([0, T_0])$ for some $T_0 > 0$, $p_0(0) = 0$; $S_0 = \{(t,x) \in D; t \leq T_0,\ x = p_0(t)\}$ is called the shock curve, and we set $S_0^\pm = \{(t,x) \in D;\ t \leq T_0,\ \pm(x - p_0(t)) > 0\}$. Concerning the function u_0, it is defined in $D \cap \{t \leq T_0\}$, it is O-valued and suffers a jump through S_0; it is of class C^1 in $S_0^+ \cup S_0$ and $S_0^- \cup S_0$. We denote by $u_0^\pm$ the restrictions of u_0 to $S_0^\pm \cup S_0$. The shock curve is not characteristic, and we assume that Lax's κ-shock condition holds, i.e.,

$$\begin{gathered}\lambda_\kappa\Big(u_0^+(t, p_0(t))\Big) < p_0'(t) < \lambda_\kappa\Big(u_0^-(t, p_0(t))\Big) \\ \lambda_{\kappa-1}\Big(u_0^-(t, p_0(t))\Big) < p_0'(t) < \lambda_{\kappa+1}\Big(u_0^+(t, p_0(t))\Big),\end{gathered} \tag{1.2}$$

for $0 \leq t \leq T_0$. We ask moreover that the shock wave is uniformly stable ([M1]), i.e.,

$$\det\big([u_0], r_1(u_0^-), \ldots, r_{\kappa-1}(u_0^-), r_{\kappa+1}(u_0^+), \ldots, r_N(u_0^+)\big)\Big|_{x=p_0(t)} \neq 0 \tag{1.3}$$

for every $t \in [0, T_0]$. Finally, u_0 is a weak solution of (1.1), i.e., denoting by $[\cdot]$ the jump through S_0,

$$
\text{(1.4)} \qquad \begin{cases} \partial_t u_0^\pm + A(u_0^\pm)\partial_x u_0^\pm = b(t, x, u_0^\pm) & \text{in } S_0^\pm \\ p_0'[u_0] - [f(u_0)] = 0 & \text{on } S_0 \\ u_0^\pm{}_{|t=0} = h_0^\pm, p_0(0) = 0, \end{cases}
$$

for some $h_0^\pm$. Let us redress this free-boundary value problem by making the change of variables $\tilde{x} = x - p_0(t)$. We still denote with x, u_0 the new (in)dependent variables, for sake of simplicity; therefore we obtain

$$
\text{(1.5)} \qquad \begin{cases} \partial_t u_0^\pm + \left(A(u_0^\pm) - p_0'\right)\partial_x u_0^\pm = b(t, x + p_0(t), u_0^\pm) & \text{in } \pm x > 0 \\ p_0'[u_0] - [f(u_0)] = 0 & on \text{x=0} \\ u_0^\pm{}_{|t=0} = h_0^\pm,\ p_0(0) = 0, \end{cases}
$$

for $t \in [0, T_0]$. We shall work with this redressed problem. So, let us define $\lambda_{k,0}^\pm = \lambda_k(u_0^\pm) - p_0'$, $r_{k,0}^\pm = r_k(u_0^\pm)$, $l_{k,0}^\pm = l_k(u_0^\pm)$, for $k = 1, \dots, N$. The propagation fields are defined by

$$
X_k^\pm = \partial_t + \lambda_{k,0}^\pm \partial_x, \qquad k = 1, \dots, N,\ \pm x \geq 0.
$$

We restrict ourselves to some interval $[-a, a]$ of the x-axis, $a > 0$, and choose T_0 sufficiently small in such a way to work in a domain of determinacy $\Omega_0^\pm \subset I_0^\pm \times [0, T_0]$ of the linearized mixed problem; we denoted $I_0^+ = [0, a]$, $I_0^- = [-a, 0]$. This means more precisely that $\Omega_0^\pm \cap \{t = 0\} = I_0^\pm$ and every characteristic curve of the propagation fields $X_k^\pm$ exists and is smooth in $\Omega_0^\pm$, unless it reaches the boundary $\{x = 0\}$ at some time $T_k^\pm < T_0$.

We associate to each field $X_k^\pm$ a phase space $\Phi_k^\pm$. It is a finite dimensional vector space consisting of C^2 functions $\varphi_k^\pm$ that satisfy $X_k^\pm \varphi_k^\pm = 0$; moreover we ask that $\nabla \varphi_k^\pm \neq 0$ a.e. if $\varphi_k^\pm \in \Phi_k^\pm \setminus \{0\}$. Another phase space Φ^0 is needed: it is spanned by the traces at $\{x = 0\}$ of the ingoing phases, i.e., by

$$
\{\varphi^+(\cdot, 0);\ \varphi^+ \in \Phi_i^+ \text{ for some } 1 \leq i \leq \kappa\} \cup \{\varphi^-(\cdot, 0);\ \varphi^- \in \Phi_i^- \text{ for some } \kappa \leq i \leq N\}.
$$

It is still a finite dimensional vector space and we ask that $d\varphi^0/dt \neq 0$ a.e. if $\varphi^0 \in \Phi^0 \setminus \{0\}$, analogously as before.

On these spaces we make the following assumptions:

(C) Closure: if $s_i^\pm \in \Phi_i^\pm$, $s_j^\pm \in \Phi_j^\pm$ and $X_k^\pm(s_i^\pm + s_j^\pm) = 0$, then $s_i^\pm + s_j^\pm \in \Phi_k^\pm$;

(T) Transversality: if $s_i^\pm \in \Phi_i^\pm$, $s_j^\pm \in \Phi_j^\pm$ and $X_k^\pm(s_i^\pm + s_j^\pm) \not\equiv 0$, then $X_k^\pm(s_i^\pm + s_j^\pm) \neq 0$ a.e. along each characteristic of $X_k^\pm$.

Analogous assumptions are required for the "boundary" phases:

(Cb) Closure under reflection-transmission at the boundary: if $X_k^\pm$ is an outgoing field, then $X_k^\pm s_k^\pm = 0$ and $s^\pm_{|t=0} \in \Phi^0$ imply $s^\pm \in \Phi_k^\pm$;

(Tb) Transversality for boundary phases: if $s_i^\pm \in \Phi_i^\pm$, $s^0 \in \Phi^0$ and $X_k^\pm(s_i^\pm + s^0) \not\equiv 0$, then $X_k^\pm(s_i^\pm + s^0) \neq 0$ a.e. along each characteristic of $X_k^\pm$.

Clearly these assumptions must hold for each $1 \le i, j, k \le N$. The dual spaces of $\Phi_k^\pm$, Φ^0, are $\Theta_k^\pm$, Θ^0, respectively; we define a $\Theta_k^\pm$-valued phase function $\vec{\varphi}_k^\pm$ through

$$\langle \vec{\varphi}_k^\pm(t,x), s^\pm \rangle = s^\pm(t,x),$$

for $s^\pm \in \Phi_k^\pm$, angle brackets meaning duality between $\Theta_k^\pm$ and $\Phi_k^\pm$. Analogously we define $\langle \vec{\varphi}^0(t), s^0 \rangle = s^0(t)$, for $s^0 \in \Phi^0$.

Let us now introduce some spaces of almost periodic functions. Let Ψ be a finite dimensional real vector space; we denote by $C_{ap}^0(\Psi)$ the space of almost periodic real functions on Ψ, i.e., the completion in the $L^\infty(\Psi)$-norm of the exponential functions $\{\exp(i\langle \lambda, \cdot \rangle)\}$, $\lambda \in \Psi^*$ (= dual space to Ψ). More generally $C_{ap}^k(\Psi)$ denotes the space of k-times differentiable almost periodic real functions on Ψ. If $\Omega \subset R_{t,x}^2$, we denote also $C^0(\Omega; \Psi) = C^0(\Omega, C_{ap}^0(\Psi))$ and analogously $C^k(\Omega; \Psi)$.

On the space $C_{ap}^0(\Psi)$ it is possible to define mean value operators that average on subspaces of Ψ: they are obvious generalizations of the usual mean value operators for almost periodic functions (averaging on the whole space). They play a key role in the following, where the relevant subspaces are associated to resonances in the phase spaces. In this case Ψ is going to be in turn a subspace of $\Theta^\pm = \Theta_1^\pm \times \ldots \times \Theta_N^\pm$, and for each $k = 1, \ldots, N$ there will be a mean value operator

$$E_k^\pm : C_{ap}^0(\Psi) \longrightarrow C_{ap}^0(\Theta_k^\pm).$$

On the other hand, when the space Ψ will appear as a subspace of Θ^0, then we need only the mean value operator E^0 averaging on the whole space Ψ. Therefore we may extend the mean value operators $E_k^\pm$ to almost periodic functions in $\Theta_k^\pm \times \Theta^0$; they will be denoted by the same letter. We refer to [C] for all that concerns resonances and these operators.

We introduce now the norms we need in the following. If $\Omega^\pm$ is a closed subset of $\Omega_0^\pm$, we define

$$\|u^\pm\|_{k,\Omega^\pm} = \sum_{\alpha \le k} \sup_{(t,x)\in\Omega^\pm} |\partial_{t,x}^\alpha u^\pm(t,x)|$$

and $\|u\|_{k,\Omega} = \max(\|u^+\|_{k,\Omega^+}, \|u^-\|_{k,\Omega^-})$. Very often the set $\Omega^\pm$ will de dropped as well as the k when $k = 0$. Moreover we define

$$\|u^\pm(t)\|_{k,\Omega^\pm} = \sum_{\alpha \le k} \sup_{x\in\Omega_t^\pm} |\partial_{t,x}^\alpha u^\pm(t,x)|$$

where $\Omega_t^\pm = \{x;\ (t,x) \in \Omega^\pm\}$, and analogously $\|u(t)\|_k$. At the boundary we need the norms

$$\|g\|_{k,t} = \sum_{j \le k} \sup_{s\in[0,t]} |g^{(j)}(s)|.$$

We refrain from giving analogous definitions for functions U depending on t, x and almost periodic in some θ-variables.

For functions $u_\varepsilon^\pm$ depending on a parameter $\varepsilon > 0$ we employ the norms

$$|u_\varepsilon^\pm|_{\varepsilon,k,\Omega^\pm} = \sum_{\alpha\le k} \varepsilon^\alpha \sup_{(t,x)\in\Omega^\pm} |\partial_{t,x}^\alpha u_\varepsilon^\pm(t,x)|.$$

We say that a family of functions $\{u_\varepsilon^\pm\}_{0<\varepsilon\le\varepsilon_0}$ is bounded (respectively $o(1)$) in $C_\varepsilon^k(\Omega^\pm)$ if the norms $|u_\varepsilon^\pm|_{\varepsilon,k,\Omega^\pm}$ are uniformly bounded for $0 < \varepsilon \le \varepsilon_0$ (resp., if $|u_\varepsilon^\pm|_{\varepsilon,k,\Omega^\pm} \to 0$ as $\varepsilon \to 0$).

At last we introduce the perturbed problem we are going to study,

$$\text{(1.6)} \qquad \begin{cases} \partial_t u^\pm + A(u^\pm)\partial_x u^\pm = b(t,x,u^\pm) & \text{in } \pm(x-p(t)) > 0 \\ p'[u] - [f(u)] = 0 & \text{on } x = p(t) \\ u^\pm{}_{|t=0} = h_0^\pm + \varepsilon h_\varepsilon^\pm, \ p(0) = 0, & \end{cases}$$

for some $h_\varepsilon^\pm$ bounded in $C_\varepsilon^1(I_0^\pm)$. As before for the background state we redress this problem by means of the change of variables $\tilde{x} = x - p(t)$. Maintaining the same notations we obtain

$$\text{(1.7)} \qquad \begin{cases} \partial_t u^\pm + (A(u^\pm) - p')\,\partial_x u^\pm = b(t, x+p(t), u^\pm) & \text{in } \pm x > 0 \\ p'[u] - [f(u)] = 0 & \text{on } x = 0 \\ u^\pm{}_{|t=0} = h_0^\pm + \varepsilon h_\varepsilon^\pm, \ p(0) = 0. & \end{cases}$$

Since the problem is quasilinear, the domain of determinacy possibly slightly differs from $\Omega_0^\pm$. Concerning (1.7) we shall then work in $\Omega_\rho^\pm$, $\rho > 0$, where if $\Omega_0^+ = \{(t,x);\ t \le T_0,\ x \le \chi^+(t)\}$ for some function χ^+, then $\Omega_\rho^+ = \{(t,x);\ t \le T_0,\ x \le \chi^+(t) - \rho t\}$, and analogously Ω_ρ^-.

So far we specified the right or left-hand part of shock waves by writing $\pm$; since both the background and the perturbed shock have been redressed, let us drop this heavy notation. However for sake of precision it will be kept in set notations and occasionally it will be used again for functions in order to avoid confusion.

We make the ansatz

$$\begin{aligned} u(t,x) &= u_0(t,x) + \varepsilon u_\varepsilon(t,x) \qquad \pm x > 0 \\ p'(t) &= p_0'(t) + \varepsilon k_\varepsilon(t) \end{aligned}$$

and plug it into (1.7). After some manipulations we obtain the problem for $(u_\varepsilon, k_\varepsilon)$:

$$\text{(1.8)} \quad \begin{cases} \partial_t u_\varepsilon + \left(A_0^\sharp(t,x,\varepsilon u_\varepsilon) - \varepsilon k_\varepsilon\right)\partial_x u_\varepsilon = b_0^\sharp(t,x,\varepsilon u_\varepsilon, \varepsilon k_\varepsilon, \varepsilon \tilde{k}_\varepsilon)(u_\varepsilon, k_\varepsilon, \tilde{k}_\varepsilon) & \text{in } \pm x > 0 \\ k_\varepsilon[u_0 + \varepsilon u_\varepsilon] - [B(t, \varepsilon u_\varepsilon)u_\varepsilon] = 0 & \text{on } x = 0 \\ u_{\varepsilon|t=0} = h_\varepsilon, & \end{cases}$$

where $\tilde{k}_\varepsilon = \int_0^t k_\varepsilon(s)ds$. The $N \times N$ matrices $A_0^\sharp$ and B are easily computed from above; for instance $A_0^\sharp(t,x,v) = A(u_0 + v) - p_0'$. The right-hand side term is computed analogously and it is linear in the $(u_\varepsilon, k_\varepsilon, \tilde{k}_\varepsilon)$ arguments.

The nonlinear integro-differential mixed problem for the profiles (U,K) of $(u_\varepsilon, k_\varepsilon)$ is

$$(1.9)\quad \begin{cases} X_k U_k + E_k\Big(\sum_{i,j}(\Gamma^k_{ij} U_i D_j U_j) - K D_k U_k\Big) \\ \qquad\qquad = E_k\Big(l_{k,0} b_0 \cdot (U, K, \tilde{K}) - l_{k,0} \sum_j (X_k r_{j,0}) U_j\Big) & \text{in } \pm x > 0 \\ K[u_0] - [A(u_0)P(u_0)U] = 0 & \text{on } x = 0 \\ U_{k|_{t=0}} = H_k, \end{cases}$$

if

$$(1.10)\qquad l_{k,0}(0,x) h_\varepsilon(x) = H_k\left(x, \frac{\vec{\varphi}_k(0,x)}{\varepsilon}\right) + o(1) \qquad \text{in } C^1_\varepsilon(I_0^\pm)$$

for some $H_k \in C^1(I_0^\pm; \Theta_k^\pm)$, $k = 1, \ldots, N$. We denoted $D_j = \partial_x \vec{\varphi}_j \cdot \partial_{\theta_j}$. In (1.9) the scalar functions Γ^k_{ij} are the so-called interaction coefficients and depend only on the background state; they are defined by

$$(1.11)\qquad \Gamma^k_{ij}(t,x) = l_{k,0}(t,x)\big(\partial_u A_0^\sharp(t,x,0) r_{i,0}(t,x)\big) r_{j,0}, \qquad 1 \le i,j,k \le N.$$

Moreover $b_0(t,x) = b_0^\sharp(t,x,0,0,0)$, $\tilde{K}(t) = \int_0^t (E^0 K)(s)ds$. At the boundary appears the matrix P: it is such that $P^{-1}AP$ is diagonal. What is most important is that the boundary conditions are given (without mentioning it, in (1.9)) on a subspace of $\Theta^\pm$ which is parametrized by $\theta_k^\pm$, for each k such that X_k is outgoing, and varies with k. It takes into account the relations among the traces of the ingoing phases on the shock curve. We refer to [C] for further details.

We look for piecewise C^1 solutions of both (1.8) and (1.9). Then two compatibility conditions between initial data and boundary conditions must be imposed to prevent the issuing of discountinuities from $(0,0)$. Here is the main result of [C]; we denote $\Omega^\pm_{\rho,T} = \Omega^\pm_\rho \cap \{t \le T\}$.

THEOREM 1.1. ([C]) *We assume conditions (C,T,Cb,Tb) on the phases and two compatibility conditions both for (1.8) and (1.9). Let h_ε be bounded in $C^1_\varepsilon(I_0^\pm)$ and satisfy (1.10). Then for every $\rho > 0$ there exist positive numbers T, ε_0 such that:*

(i) problem (1.8) has a unique solution $(u_\varepsilon^\pm, k_\varepsilon)$ which is bounded in $C^1_\varepsilon(\Omega^\pm_{\rho,T}) \times C^1_\varepsilon([0,T])$, for $0 < \varepsilon \le \varepsilon_0$;

(ii) problem (1.9) has a unique solution $(U^\pm, K)$, $U_k^\pm \in C^1(\Omega^\pm_{\rho,T}; \Theta_k^\pm)$, $K \in C^1([0,T]; \Theta^0)$;

(iii) the following asymptotic expansions hold:

$$(1.12)\qquad u_\varepsilon^\pm(t,x) - \sum_{k=1}^N U_k^\pm\left(t, x, \frac{\vec{\varphi}_k(t,x)}{\varepsilon}\right) r^\pm_{k,0}(t,x) = o(1) \qquad \text{in } C^1_\varepsilon(\Omega^\pm_{\rho,T})$$

$$(1.13)\qquad k_\varepsilon(t) - K\left(t, \frac{\vec{\varphi}^0(t)}{\varepsilon}\right) = o(1) \qquad \text{in } C^1_\varepsilon([0,T]).$$

The problem we want to consider in this paper regards the life span of the solutions $(u_\varepsilon, k_\varepsilon)$, (U, K) of (1.8), (1.9) and, as a consequence, until which times an asymptotics like (1.12), (1.13) holds. From [JMR1] it is known that, even for semilinear systems, a solution u_ε may exists for every positive time, for every $\varepsilon > 0$, while the profile blows up at finite time. This occurs when there is no uniform control on the L^∞ norms of the family u_ε. In the quasilinear case the same phenomenon is produced when the boundedness of $\varepsilon\|\nabla u_\varepsilon\|$ fails. Therefore, following [JMR1], we define, for $\delta > 0$

$$T_*(\delta) = \sup\{T \in]0, T_0];\ (1.8) \text{ has solution } (u_\varepsilon, k_\varepsilon) \text{ bounded in } C^1_\varepsilon(\Omega^\pm_{\rho,T}), \text{ for } 0 < \varepsilon \leq \delta\}.$$

Here and in the following the inessential shrinking constant $\rho > 0$ is supposed fixed once for all. Clearly $\delta \mapsto T_*(\delta)$ is a nonincreasing function. Let us call

$$T_* = \sup\{T \in]0, T_0];\ (1.9) \text{ has solution } (U, K) \text{ in } C^1(\Omega^\pm_{\rho,T}; \Theta^\pm) \times C^1([0, T]; \Theta^0)\}.$$

Then we have the following result.

THEOREM 1.2. *Under the assumptions of Theorem 1.1 we have*

$$T_* = \lim_{\delta \to 0} T_*(\delta).$$

Moreover the asymptotic expansions (1.12), (1.13) hold in $\Omega^\pm_{\rho,T}$, $[0, T]$, respectively, for every $T < T_*$, $0 < \varepsilon \leq \varepsilon_0(T)$.

The proof is given in Section 3.

2. One-dimensional gasdynamics

In this section we write explicitly the equations for the profiles in the case of one-dimensional gasdynamics. A special emphasis is accorded to the boundary conditions that were rather disregarded in the previous section.

The system of one-dimensional gasdynamics is

$$(2.1)\qquad \begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho v) + \partial_x(p + \rho v^2) = 0 \\ \partial_t\big(\rho(v^2/2 + e)\big) + \partial_x\big(\rho v(v^2/2 + e) + pv\big) = 0. \end{cases}$$

Notations are as follows: ρ is the mass density, v the flow velocity, $p = p(\rho, S)$ the pressure, S the entropy, $e = e(\rho, S)$ the internal energy per unit mass. Moreover $c = (p_\rho)^{1/2}$ will stand for the speed of sound. There is a link between ρ, p, S, e and the temperature $T = T(\rho, S)$, given by the second law of thermodynamics

$$TdS = de + pd\rho^{-1}.$$

We consider the simplest case of a polytropic gas, i.e., $p = R\exp(S/c_v)\rho^\gamma$ for some constants R, c_v and $1 < \gamma < 2$.

The system (2.1) is strictly hyperbolic with eigenvalues

$$\lambda_1 = v - c, \qquad \lambda_2 = v, \qquad \lambda_3 = v + c,$$

and right eigenvectors $r_1 = {}^t(\rho, -c, 0)$, $r_2 = {}^t(p_S, 0, -c^2)$, $r_3 = {}^t(\rho, c, 0)$; the left eigenvectors l_k are deduced by the normalization condition. The first and third eigenvalue are genuinely nonlinear.

We take as background state a piecewise constant 1-shock. This means that the shock curve is the line $x = \sigma_0 t$, $\sigma_0 \in R$, and Lax's condition becomes

$$v_0^+ - c_0^+ < \sigma_0 < v_0^- - c_0^-, \qquad \sigma_0 < v_0^+.$$

On the left-hand side all of the three characteristic are ingoing; on the right the first one does, while the two others are outgoing. We assume that this shock is uniformly stable. We call $u = {}^t(\rho, v, S)$, $u_0^\pm = {}^t(\rho_0^\pm, v_0^\pm, S_0^\pm)$.

System (2.1) is not written under the form (1.1) but rather as

$$\partial_t(g(u)) + \partial_x(f(u)) = 0$$

with $g = {}^t(\rho, \rho v, \rho(v^2/2 + e))$, $f(u) = {}^t(\rho v, p + \rho v^2, \rho v(v^2/2 + e) + pv)$. This means that in the computation of the interaction coefficients Γ_{ij}^k the matrix $B^{-1}A$ takes the place of the matrix A in (1.11), where B is the Jacobian matrix of g. The perturbed problem we are going to consider is then

$$\text{(2.2)} \qquad \begin{cases} \partial_t\left(g(u)\right) + \partial_x\left(f(u)\right) = 0 & \text{in } \pm(x - \sigma(t)) > 0 \\ \sigma'\left[g(u)\right] - \left[f(u)\right] = 0 & \text{on } x - \sigma(t) = 0 \\ u_{|t=0} = u_0 + \varepsilon h_\varepsilon, \; \sigma(0) = 0, \end{cases}$$

with h_ε as in Theorem 1.1. We assume that some phase spaces satisfying conditions *(C,T,Cb,Tb)* are given; we suppose moreover that they are of dimension 1. As in the previous section we perform the change of variables $\tilde{x} = x - \sigma(t)$ and obtain then

$$\begin{cases} B(u)\partial_t u + \left(A(u) - \sigma'\right)\partial_x u = 0 & \text{in } \pm x > 0 \\ \sigma'\left[g(u)\right] - \left[f(u)\right] = 0 & \text{on } x = 0 \\ u_{|t=0} = u_0 + \varepsilon h_\varepsilon, \; \sigma(0) = 0. \end{cases}$$

We make the ansatz $u = u_0 + \varepsilon u_\varepsilon$ in $\pm x > 0$, and $\sigma'(t) = \sigma_0 + \varepsilon k_\varepsilon(t)$. Then Theorem 1.1 implies

$$u_\varepsilon^\pm(t,x) = \sum_{j=1}^{3} U_j\left(t, x, \frac{\vec{\varphi}_j(t,x)}{\varepsilon}\right) r_j(u_0) + o(1) \qquad \pm x > 0, \text{ in } C_\varepsilon^1$$

$$k_\varepsilon(t) = K\left(t, \frac{\vec{\varphi}^0(t)}{\varepsilon}\right) + o(1) \qquad \text{in } C_\varepsilon^1,$$

where the interior equations for the profiles are

$$(2.3)\quad \begin{cases} X_1U_1 + (\alpha_1U_1 + \alpha_2\overline{U}_2 + \alpha_3\overline{U}_3 - \overline{K})D_1U_1 + \frac{1}{2}\alpha_4E_1(U_3D_2U_2) = 0 \\ X_2U_2 - (c_0\overline{U}_1 - c_0\overline{U}_3 + \overline{K})D_2U_2 = 0 \\ X_3U_3 - (\alpha_1U_3 + \alpha_2\overline{U}_2 + \alpha_3\overline{U}_1 + \overline{K})D_3U_3 - \frac{1}{2}\alpha_4E_3(U_1D_2U_2) = 0, \end{cases} \qquad \pm x > 0.$$

The vector fields X_k occurring in (2.3) are defined by

$$X_1 = \partial_t + (v_0 - c_0 - \sigma_0)\partial_x, \quad X_2 = \partial_t + (v_0 - \sigma_0)\partial_x, \quad X_3 = \partial_t + (v_0 + c_0 - \sigma_0)\partial_x, \qquad \pm x > 0.$$

Moreover we denoted by $\overline{V}$ the usual mean of an almost periodic function V. In (2.3) α_j, $j = 1, \ldots, 4$ stand for the interaction coefficients Γ^k_{ij}; they are calculated in [MR] for a null background flow velocity but their expression is the same here, owing to the Galilean invariance of the equations. More precisely

$$\alpha_1 = -\frac{(\gamma+1)c_0}{2}, \quad \alpha_2 = -\alpha_4 = \frac{(\gamma-1)c_0^3}{2\gamma R}, \quad \alpha_3 = \frac{(3-\gamma)c_0}{2}.$$

To show how the mean value operators act one needs to know the phase spaces. Suppose for instance that $\Phi^\pm_k$, $k = 1, 2, 3$, are the linear span of the phases $\varphi^\pm_1(t,x) = x - (v^\pm_0 - c^\pm_0 - \sigma_0)t$, $\varphi^\pm_2(t,x) = x - (v^\pm_0 - \sigma_0)t$, $\varphi^\pm_3(t,x) = x - (v^\pm_0 + c^\pm_0 - \sigma_0)t$, respectively, and then Φ^0 is the linear span of the function t. Then it is easy to see that assumptions *(C,T,Cb,Tb)* are satisfied. Moreover these phases are resonanting, i.e., they are linked by the relation

$$\varphi^\pm_2 = \frac{1}{2}\left(\varphi^\pm_1 + \varphi^\pm_3\right).$$

In this case the mean value operators are defined as

$$(E_1U)(\theta_1) = \lim_{T\to\infty}\frac{1}{T}\int_0^T U\left(\theta_1, \frac{\theta_1+\theta_3}{2}, \theta_3\right)d\theta_3$$

$$(E_3U)(\theta_3) = \lim_{T\to\infty}\frac{1}{T}\int_0^T U\left(\theta_1, \frac{\theta_1+\theta_3}{2}, \theta_3\right)d\theta_1.$$

Remark that there is no E_2 averaging operator in (2.3).

On the other hand, if we take the squares of the above phase functions and repeat the construction of the phase spaces, then assumption *(T)* fails. Nevertheless it is possible to cope with this case and obtain results analogous to the former ones, by allowing "L^p errors" (see [C]). What is important here is that these phases are nonresonanting, and now the operators E_k only average with respect to the fast variables different from θ_k. In this case therefore the last term in the left-hand side of the first and third equation of (2.3) vanish; in fact, for instance, $E_1(U_3D_2U_2) = \overline{U_3}\,\overline{D_2U_2} = 0$, the mean value of a derivative being zero.

Remark that by Poincaré theorem (Theorem 3.1.5 in [JMR1]) three vector fields have at most one resonance (modulo constants) and possibly none. Thus the two cases discussed above are essentially the only possible ones.

System 2.3 can be put in conservation form; by using some computation rules for the averaging operators one finds

$$
(2.4)\quad \begin{cases} X_1U_1 + D_1\left\{\alpha_1 U_1^2/2 + \left(\alpha_2\overline{U}_2 + \alpha_3\overline{U}_3 - \overline{K}\right)U_1 + \alpha_4 E_1(U_2U_3)\right\} = 0 \\ X_2U_2 + D_2\left\{\left(-c_0\overline{U}_1 + c_0\overline{U}_3 - \overline{K}\right)U_2\right\} = 0 \\ X_1U_3 - D_3\left\{\alpha_1 U_3^2/2 + \left(\alpha_2\overline{U}_2 + \alpha_3\overline{U}_1 + \overline{K}\right)U_3 - \alpha_4 E_3(U_2U_1)\right\} = 0, \end{cases} \qquad \pm x > 0.
$$

Let us now turn to the boundary conditions. Recall that the outgoing components of the profiles are U_2^+, U_3^+; this means that in the iterative scheme solving (2.3) only these components (and K) will need the boundary conditions since the ingoing components $U_1^-, U_2^-, U_3^-, U_1^+$ are completely determined by the initial data. So let us write the boundary conditions under the form

$$
(2.5)\qquad \mathcal{S}_0{}^t(K, U_2^+, U_3^+) = \mathcal{T}_0{}^t(U_1^-, U_2^-, U_3^-, U_1^+) \qquad \text{on } x = 0,
$$

where

$$
\mathcal{S}_0 = \begin{pmatrix} [\rho] & -\sigma(p_S)^+ & -\rho^+(\sigma - c^+) \\ [\rho v] & 0 & -c^+\rho^+(\sigma - c^+) \\ [\rho(v^2/2 + e)] & \sigma\left(c^2\rho T - hp_S\right)^+ & -\rho^+h^+(\sigma - c^+) \end{pmatrix}_0
$$

and

$$
\mathcal{T}_0 = \left(\begin{pmatrix} \rho(\sigma + c) & \sigma p_S & \rho(\sigma - c) \\ -c\rho(\sigma + c) & 0 & c\rho(\sigma - c) \\ h\rho(\sigma + c) & -\sigma\left(c^2\rho T - hp_S\right) & h\rho(\sigma - c) \end{pmatrix}^- \begin{pmatrix} -\rho(\sigma + c) \\ c\rho(\sigma + c) \\ -h\rho(\sigma + c) \end{pmatrix}^+ \right)_0 .
$$

The subscript "0" means computations at the background state. The matrix $\mathcal{S}_0$ is not singular since the background shock is supposed uniformly stable.

We can now explain which role play the dual fast variables θ at the boundary in this simple case (one-dimensional phase spaces). Let us suppose again that the phase spaces are the span of the linear phases as above. Then at the boundary we have the relations

$$
(2.6)\qquad \varphi_i(0,t) = \frac{\lambda_i(u_0)}{\lambda_j(u_0)}\varphi_j(0,t)
$$

for $i, j = 1, 2, 3$. We omitted the $\pm$ signs, since (2.6) clearly hold for whatever choice of the signs $+$ or $-$ at the left or right-hand side. These relations, as well as the analogous ones for the boundary phase, are reflected in the dual fast variables by means of transposition, i.e.,

$$
\theta_i = \frac{\lambda_j(u_0)}{\lambda_i(u_0)}\theta_j, \quad \theta_i = -\frac{1}{\lambda_{i,0}}\theta^0, \qquad \text{on } x = 0.
$$

We have written $\lambda_{i,0} = \lambda_i(u_0) - \sigma_0$. Therefore we can compute explicitly K, U_2^+, U_3^+ in (2.5) in the following way:

$$K(\theta^0) = {}^t e_1\,(\mathcal{S}_0^{-1}\mathcal{T}_0)\ {}^t\Big((U_1^-(-\frac{1}{\lambda_{1,0}^-}\theta^0), U_2^-(-\frac{1}{\lambda_{2,0}^-}\theta^0), U_3^-(-\frac{1}{\lambda_{3,0}^-}\theta^0), U_1^+(-\frac{1}{\lambda_{1,0}^+}\theta^0)\Big)$$

$$U_j^+(\theta_j^+) = {}^t e_j\,(\mathcal{S}_0^{-1}\mathcal{T}_0)\ {}^t\Big(U_1^-\Big(\frac{\lambda_1^-}{\lambda_j^+}\theta_j^+\Big), U_2^-\Big(\frac{\lambda_2^-}{\lambda_j^+}\theta_j^+\Big), U_3^-\Big(\frac{\lambda_3^-}{\lambda_j^+}\theta_j^+\Big), U_1^+\Big(\frac{\lambda_1^+}{\lambda_j^+}\theta_j^+\Big)\Big)$$

for $j = 2,3$. In the second line we wrote for more precision $\lambda_j^\pm = \lambda_j(u_0^\pm)$. At last, we come back to the old variables of (2.2) and write the asymptotic expansion we found:

$$u_\varepsilon^\pm(t, x-\sigma(t)) = \sum_{k=1}^{3} U_k^\pm\left(t, x-\sigma(t), \frac{\varphi_k^\pm(t, x-\sigma(t))}{\varepsilon}\right) r_{k,0} + o(1) \quad \text{in } C_\varepsilon^1,\ \pm(x-\sigma(t)) > 0$$

$$\sigma(t) = \sigma_0 t + \varepsilon\int_0^t \overline{K}(s)ds + o(\varepsilon) \quad \text{in } C_\varepsilon^2, \qquad \sigma'(t) = \sigma_0 + \varepsilon K\left(t, \frac{\varphi^0(t)}{\varepsilon}\right) + o(1) \quad \text{in } C_\varepsilon^1.$$

3. Proof of Theorem 1.2.

In this section we refer constantly to [C]; every formula identified by three numbers ((1.4.20), for instance) and appearing there is cited here without quoting explicitly that paper. The techniques which are going to be used are those exploited in [JMR1] and [C]. The result we prove is slightly more general than that stated in Theorem 1.2, since it allows phase shifts (with dual variable τ) and requires weaker hypotheses on the phases than those we made there.

We consider the problems (1.8), (1.9) as written under the form (1.4.20), (1.4.21). From Theorem 1.1 we know that there exists ε_1 and a time T_{th} (th for theorem) such that the solutions $(u_\varepsilon, k_\varepsilon), (U, K)$ exist until this time, for $\varepsilon \le \varepsilon_1$, and the asymptotics holds. Moreover it is easy to see that

$$T_{th} \ge \inf\Big\{T_0, t(\mu, \mu(\varepsilon_0))\Big\}$$

for some function $t(\cdot,\cdot)$, where $\mu = \|H\| + \|\nabla H\|$, $\mu(\varepsilon_0) = \sup_{0<\varepsilon\le\varepsilon_0}(\|h_\varepsilon\| + \varepsilon\|\nabla h_\varepsilon\|)$.

As a first step we prove that $T_* \le \lim_{\delta\to 0} T_*(\delta)$ following [JMR1]. Let us fix $T < T_*$; we should find some $\delta > 0$ such that $T < T_*(\delta)$. We define $M = \sup(\|U\|_1, \|K\|_1)$ and call $T_M = t(M, M+1)$. Since of (1.10), $\mu(\delta_1) \le M+1$; therefore $T_{th} \ge \inf(T_0, T_M)$ and applying Theorem 1.1 we see that there exists $\varepsilon_1 \in]0, \delta_1]$ in order that $T_*(\varepsilon_1) \ge t_1 = \inf(T_0, T_M)$. If now $t_1 > T$, we have finished; otherwise, since

$$\|U(T_M, \cdot, \cdot)\|_1 \le M,$$

then

$$\|u_\varepsilon(T_M)\| + \varepsilon\|\nabla u_\varepsilon(T_M)\| \le M+1 \qquad 0 < \varepsilon \le \delta_2,$$

for some δ_2. We apply again Theorem 1.1 but with initial data at $t = T_M$; we are allowed to do this since the compatibility conditions both for the exact and the profile problem are trivially satisfied at $t = T_M$. Then we have a further gain T_M of life span and after a finite number of steps we arrive at some $\delta > 0$ such that $T_*(\delta) > T$. Remark that neither k_ε nor K appear in the proof: only u_ε and U have initial data.

To prove that $T_* \geq \lim_{\delta\to 0} T_*(\delta)$ we need the following lemma.

LEMMA 3.1. *Define $m = \|H\| + \|DH\|$. Then there exists a positive function $t(\cdot)$ such that for $T = \inf\{T_0, t(m)\}$ problem (1.9) has a solution $(U, K) \in C^1(\Omega^\pm_{\rho,T}; \Psi^\pm) \times C^1([0,T]; \Psi^0)$.*

Proof. Recall the iterative scheme $(2.2.21)_{\nu+1}$ that was employed to solve problem (1.9), and rewrite it by using (2.2.16). If we denote $m_\nu(t) = \|U^\nu(t)\| + \|DU^\nu(t)\|$, then by Proposition 2.2.1

$$\|U^{\nu+1}(t)\| \leq C\left(\|H + \int_0^t m_\nu(s)\,(1 + \|U^{\nu+1}(s)\|)\,ds\right) \tag{3.1}$$

for some constant C. Let us call $(Z_k^{\nu+1}, Q^{\nu+1}) = (-\lambda_{k,0} D_k U_k^{\nu+1}, D^0 K^{\nu+1})$. Writing the system satisfied by $(Z_k^{\nu+1}, Q^{\nu+1})$ and applying once again Proposition 2.2.1 we see that

$$\|DU^{\nu+1}(t)\| \leq C\left(\|DH\| + \int_0^t m_\nu(s)\left((1 + \|DU^{\nu+1}(s)\|) + \|DU^{\nu+1}(s)\|\right) ds\right). \tag{3.2}$$

Adding (3.1) to (3.2) and using Gronwall's inequality we find

$$m_{\nu+1}(t) \leq (Cm+1)e^{C\int_0^t (m_\nu(s)+1)ds} - 1.$$

Then, by induction on ν,

$$m_\nu(t) \leq C\frac{m + (Cm+1)t}{1 - C(Cm+1)t}$$

for $t < 1/[C(Cm+1)]$ and therefore

$$m_\nu(t) \leq 2Cm \tag{3.3}$$

for $t \leq m/[(Cm+1)(2Cm+1)]$. With the help of estimate (3.3) we claim that the sequence $\{(U^\nu, K_\nu)\}$ is uniformly bounded in $C^1(\Omega^\pm_{\rho,t(m)}; \Psi^\pm) \times C^1([0, t(m)]; \Psi^0)$, where $t(m) = [1/2C]\log(1 + 1/m)$. In fact, let us call $Z^{\nu+1} = \partial_{t,\theta,\tau} U^{\nu+1}, Q^{\nu+1} = \partial_{t,\theta^0,\tau^0} K^{\nu+1}$. Then $(Z^{\nu+1}, Q^{\nu+1})$ is solution of a system as

$$X_k Z_k^{\nu+1} + \left(\mu_k(U^\nu) - E^0(K^\nu)\right) D_k Z_k^{\nu+1} = G_k^{\nu+1} \qquad \pm x > 0, \tag{3.4}$$

coupled with some boundary conditions and with initial data that are deduced in part from the interior equations. The right-hand side term $G^{\nu+1}$ in (3.4) is the sum of linear term in $Z_k^{\nu+1}, U^\nu, K^\nu, \tilde{K}^\nu, Z^\nu, Q^\nu$ and bilinear terms containing $U^\nu \cdot Z^{\nu+1}, Z^\nu \cdot DU^{\nu+1}, Q^\nu \cdot DU^{\nu+1}, K^\nu \cdot Z^{\nu+1}, U^{\nu+1} \cdot Z^\nu, DU^\nu \cdot Z^{\nu+1}$, where dot means product of some components of the vector functions under consideration. For this problem one compatibility condition is satisfied as a consequence of the assumptions of Theorem 1.1; so we can apply again Proposition 2.2.1 and using (3.3) we obtain (dot notation as above)

$$\|Z^{\nu+1}(t)\| \leq C\left(\|H \cdot DH\| + m + (1+m)\int_0^t \left(\|Z^{\nu+1}(s)\| + \|Z^\nu(s)\| + 1\right) ds\right)$$

for $t < m/[(Cm+1)(2Cm+1)]$ and analogous estimates for $\|Q^{\nu+1}(t)\|$. An easy induction on ν proves then the claim.

Finally, the continuity moduli are estimated as in [JMR1] or [C] and so an application of Ascoli's theorem concludes the proof of the lemma.

Let us now show how to complete the proof of Theorem 1.2. We assume by contradiction that $T_* < T_*(\delta)$ for some $\delta > 0$. Then there is a constant C such that

$$\sup\left(|u_\varepsilon|_{\varepsilon,1,\Omega_{\rho,T_*}}, |k_\varepsilon|_{\varepsilon,1,[0,T_*]}\right) \le C. \tag{3.5}$$

From the first part of the proof we have, for any $T < T_*$,

$$\varepsilon\partial_t u_{\varepsilon,k}(t,x) + \lambda_{k,0} D_k U_k\left(t, x, \frac{\vec{\varphi}_k(t,x)}{\varepsilon}, \frac{1}{\varepsilon}\right) + o(1) \qquad \text{in } C^0_\varepsilon(\Omega^\pm_{\rho,T}) \tag{3.6}$$

(see (2.3.43)). The theorem of uniqueness of profiles (Theorem 2.10.5 in [JMR1]) then implies together with (3.5), (3.6) that

$$\|U(t)\| + \|(\lambda_{k,0} DU)(t)\| \le M$$

and therefore $\|U(t)\| + \|DU(t)\| \le M'$ for some M', $t \le T$. Lemma 2.1 applied at $t = T$ then assures that U is defined at least until time $T + t(M')$. If T is sufficiently near to T_* this is a contradiction. Theorem 1.2 is then completely proved.

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